Towards QUBO formulation of WMN joint link scheduling and routing for quantum computers

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Introduction

Link scheduling and routing in Wireless Mesh Networks (WMN) are essential problems in optimal resource utilization. To find an optimal solution, link scheduling and routing should be solved as a joint problem but this is challenging as link scheduling alone requires solving maximum weighted independent set (MWIS) problem, which is known to be NP-hard.

Here we consider a millimeter wave (mmW) WMN that operates in E band (70–90 GHz), which enables us to use *"pseudowire"* assumption (interference can be mostly ignored). The system operates in TDM mode with fixed-length time slots. There are some physical and technical constraints (depending on node type) that define which links can be active at the same time.

The optimization goal is to minimize worst case edge-to-edge delays. In earlier work we have solved link scheduling and routing problem with heuristic classical algorithms [4, 1] but here we are looking for reformulating the problem for NISQ devices.

Joint link scheduling and routing

Our idea of solving joint link scheduling and routing problem is to turn the problem in to a single graph problem, which is possible if the schedule length is a fixed value. We present problem as a graph that mixes both spatial links (radio links between nodes) and time links (time spend in traveling a link or waiting for transmission opportunity). The main task is to find a set of spatial links that fulfill the link scheduling constraints and provide optimal "time-space" paths between gateway (GW) and non-GW nodes.

Generating "Time-Space" graph

Given WMN topology $G = (V, E)$ and schedule length L construct "time-space graph" H (digraph) with set of schedule slots $S_s = [1, L]$

- $V(H) = V(G) \times S_s$, $u_{(i,k)} \in V(H)$ where $u_{(i,k)} \sim u_i$ in schedule slot k
- $T_k = \{(u_{(i,k)}, u_{(j,k)}) | (u_i, u_j) \in E(G)\}, k \in S_s$ edges in time-slot k
- $D(H) = \{(u_{(i,k)}, u_{(i,k+1 \bmod L)}) | k \in S_s\}$ time edges between time-slots
- $E(H) = \bigcup_k T_k \cup D(H)$ (all T_k + edges between time-slots)

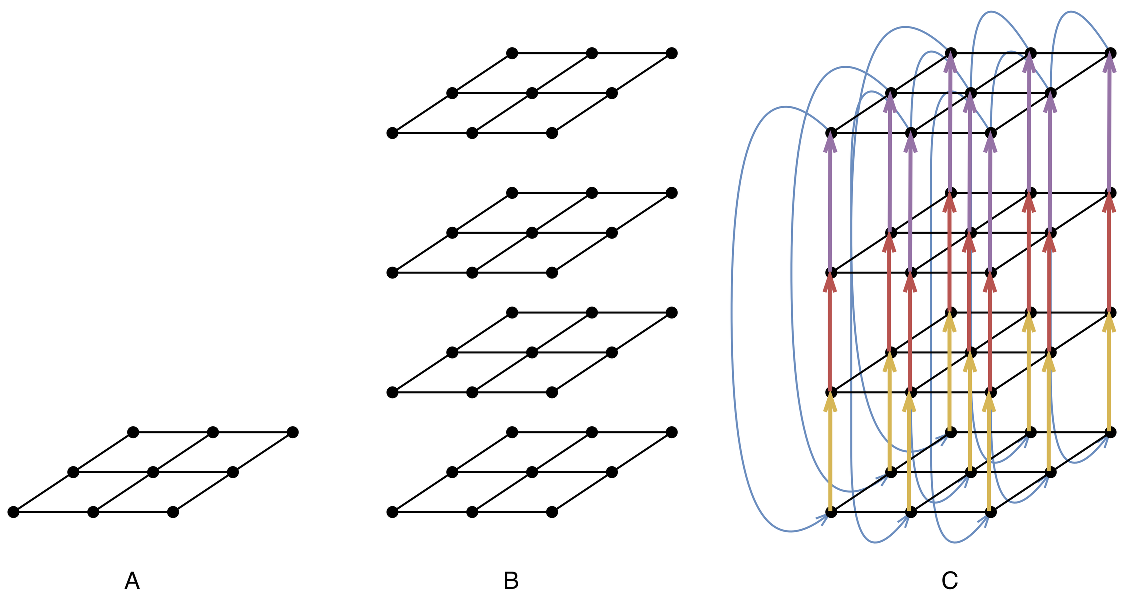

Figure 1: Constructing "time-space" graph — A) the original network topology, B) making copies of original topology for each time-slot, C) adding time links

To make it easier to find optimal routing and scheduling, few extra nodes are added

- Gateway terminals for TX and RX $u_{g,tx}, u_{g,rx}$ to $V(H)$
- Given set of GW nodes $S_g \in V(G)$, add edges $(u_{g,tx}, u_{(i,k)}), (u_{(i,k)}, u_{g,rx}) | u_i \in S_g, k \in S_s$ to $E(H)$
- Destination terminals for TX and RX $u'_{i,tx}, u'_{i,rx} | \forall u_i \notin S_g$ to $V(H)$
- Edges $(u'_{i,tx}, u_{(i,k)}), (u_{(i,k)}, u'_{i,rx}) | k \in S_s, u_i \notin S_g$ to $E(H)$

Link scheduling constraints

Link scheduling constraints (it is assumed that single WMN contains only one type of nodes):

- Single-RU (radio unit) case: in each T_k each u should have at most one active link with any $v \in T_k | v \neq u$, so that both (u, v) and (v, u) are active
- Multi-RU case: in each T_k each u should have either in-links or out-links only with any $v \in T_k | v \neq u$

Optimization problem as Steiner tree problem

To search for optimal solution, find a set of spatial links $\in V(H)$ that so that H provides optimal paths a) from $u_{g,tx}$ to all $u'_{i,rx}$ and b) from all $u'_{i,tx}$ to $u_{g,rx}$ fulfilling link scheduling constraints.

Axiom: given link weights $w_i \geq 0$, $\exists$ tree T that gives all shortest paths from $u_{g,tx}$ to all $u'_{i,rx}$. Thus we can handle this problem as finding Steiner tree between terminals $u_{g,tx}$, all $u'_{i,rx}$. For general Steiner tree problem, QUBO formulation is given in [3]. In this case, H is digraph and as $u_{g,tx}$ has only out-links and $u'_{i,rx}$ in-links, which means that there should be no need for additional penalty terms to ensure desired root location or that destination terminals are leaf nodes. As we want to optimize worst case delays in both directions, separate Steiner trees are needed for downstream and upstream traffic flows.

It should be noted that QUBO formulation given in [3] is for undirected graph and thus it can be considered to be a bit complicated for directed graphs. There are some classical algorithms that can handle Steiner tree problem in directed (and rooted) graphs more efficiently, e.g., in polynomial time. However, finding their applicability for QUBO formulation needs some further research. Some of parameterized algorithms seem to work only with chordal graphs and in general, looks like practical solutions can be found only for limited number terminals (FPT W[2]).

Anyhow, we are looking forward to be able to adapt the QUBO formulation for digraphs by removing some of the variables, e.g., depth variable as it would be redundant with directed edges. Furthermore, we are inspecting how reduce the number of penalty terms as some of the constraints are implied by the graph topology, e.g., root location. Finally, we are considering if the tree structure constraints could be formulated to operate with edges only as in shortest path problem, e.g., [2].

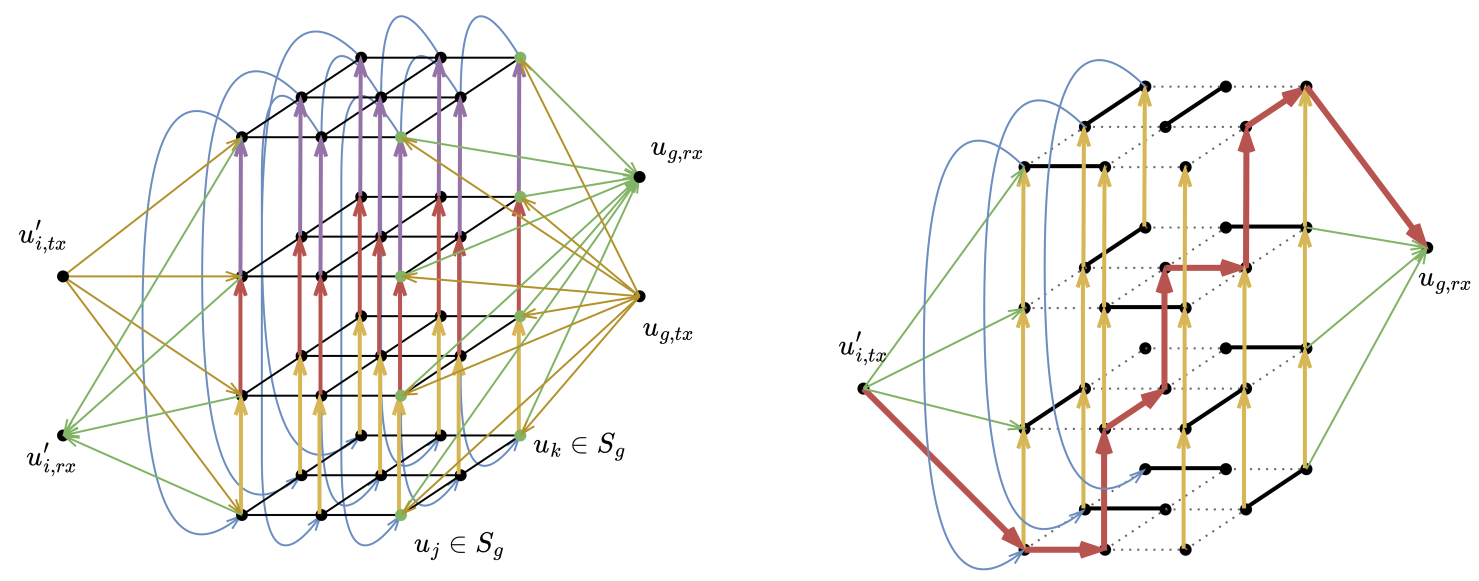

Figure 2: Left: "time-space" graph extended with target terminals (only one $u'_{i,tx/rx}$ pair shown for clarity). Right: Solving optimal link schedule and path from $u'_{i,tx}$ to $u_{g,rx}$ by finding the shortest path over set of active links.

QUBO formulation

The exact QUBO formulation is still much under construction, but the main objective function to minimize will be cost of the links included in the both Steiner trees, so that selected links: a) form proper Steiner tree between terminals b) do not violate link scheduling constraints. We are also investigating how to set link weights.

Complexity estimate

If we can reformulate the problem so that only one binary variable is needed for each edge, then considering a single Steiner tree and assuming average vertex degree of 3, each vertex would have 5 edges in total (out-links, time link, and link to destination terminal). Furthermore, there are L copies of G and thus total number of edges should be $5|V(G)|L$ and so for both trees $10|V(G)|L$. Thus the search space should be order of $2^{10|V(G)|L}$.

Classical processing

After the Steiner trees for optimal link scheduling are found, there will be few steps on classical side to complete the link scheduling and routing process. First, all T_k are checked if they contain maximal transmission set (maximal matching). If not, then additional edges are added to active link set. Finally, the subtrees starting form GW nodes are expanded to multiple spanning tree structure as in [4] to provide disjoint paths.

Acknowledgements

Our study is funded by Business Finland, see `https://www.cohqca.fi/` for further information. Companies in the project steering group are Nokia Bell Labs, Unitary Zero Space and Cumucore.

Unitary Zero Space   Cumucore   VTT

Summary

- **We have formulated the joint link scheduling and routing optimization problem as finding a Steiner tree on a time-space graph with a parameterization that the root node is fixed.**
- **We are working on adapting QUBO formulation for directed graphs.**

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