Solving Wireless Mesh Network Joint Link Scheduling and Routing Problem with Hybrid Quantum-Classical Algorithm

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Introduction

Link scheduling and routing in Wireless Mesh Networks (WMN) are essential problems in optimal resource utilization. To find an optimal solution, link scheduling and routing should be solved as a joint problem but this is challenging as link scheduling alone requires solving maximum weighted independent set (MWIS) problem, which is known to be NP-hard.

The optimization goal is to minimize worst case edge-to-edge delays. In earlier work we have solved link scheduling and routing problem with heuristic classical algorithms [4, 1] but here we are looking for reformulating the problem for NISQ devices.

Joint link scheduling and routing

Our idea of solving joint link scheduling and routing problem is to turn the problem in to a single graph problem, which is possible if the schedule length is a fixed value. We present problem as a graph that mixes both spatial links (radio links between nodes) and time links (time spend in traveling a link or waiting for transmission opportunity). The main task is to find a set of spatial links that fulfill the link scheduling constraints and provide optimal ”time-space” paths between gateway (GW) and non-GW nodes.

Generating ”Time-Space” graph

Given WMN topology $G = (V_G, E_G)$ and schedule length L construct ”time-space graph” H (digraph) with set of schedule slots $S = \{s_0, s_1, \dots, s_{L-1}\}$

- $V_H = V_G \times S_s$ where $u_{(i,k)} = (u_i, s_k) \sim u_i$ in schedule slot k
- $T_k = \{(u_{(i,k)}, u_{(j,k)}) | (u_i, u_j) \in E_G\}$ edges in time-slot k
- $D_H = \{(u_{(i,k)}, u_{(i,k+1 \bmod L)}) | k \in S_s\}$ time edges between time-slots
- $E_H = \bigcup_k T_k \cup D_H$ (all T_k + edges between time-slots)

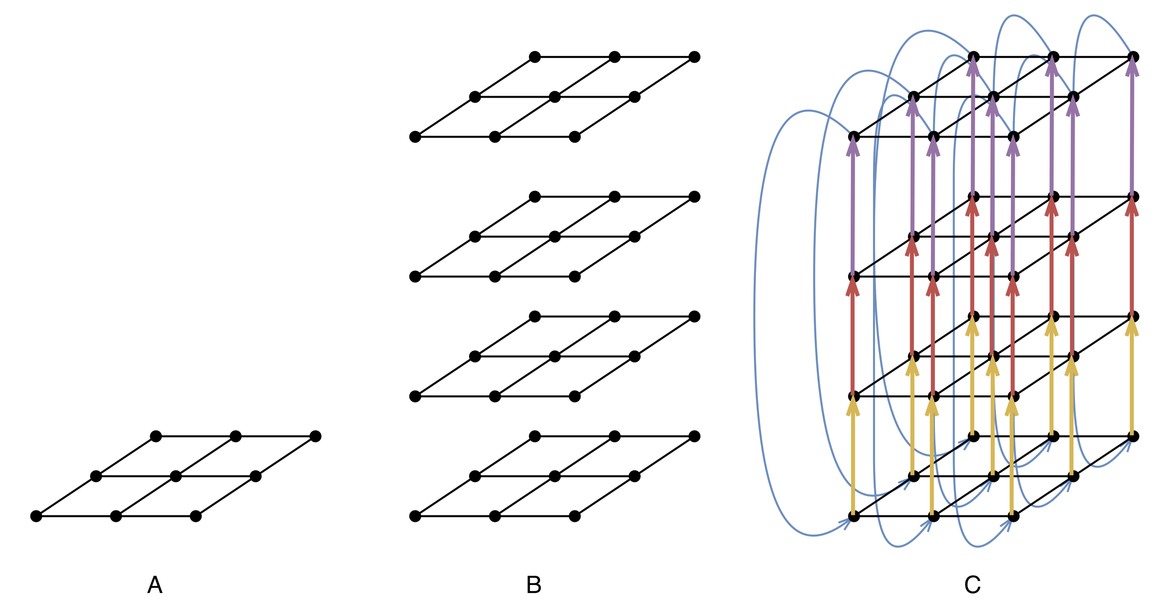

Figure 1: Constructing ”time-space” graph — A) the original network topology, B) making copies of original topology for each time-slot, C) adding time links

To make it easier to find optimal routing and scheduling, few extra nodes are added

- Gateway terminals for TX and RX u_g^{tx}, u_g^{rx} to V_H
- Given set of GW nodes $S_g \in V_G$, add edges $(u_g^{tx}, u_{(i,k)}), (u_{(i,k)}, u_g^{rx}) | u_i \in S_g, k \in S_s$ to E_H
- Destination terminals $S_d^{tx/rx}$ for TX and RX $u_i^{tx}, u_i^{rx} | \forall u_i \notin S_g$ to V_H
- Edges $(u_i^{tx}, u_{(i,k)}), (u_{(i,k)}, u_i^{rx}) | k \in S_s, u_i \notin S_g$ to E_H

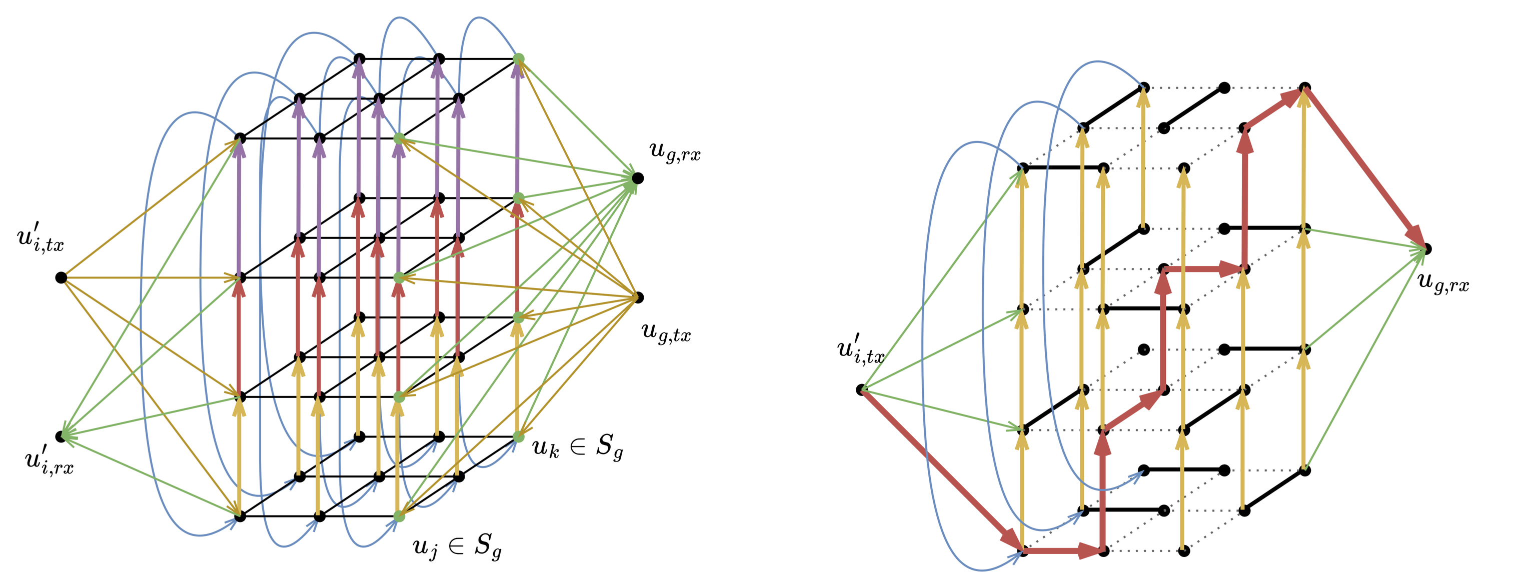

Figure 2: Left: ”time-space” graph extended with target terminals (only one $u'_{i,tx/rx}$ pair shown for clarity). Right: Solving optimal link schedule and path from $u'_{i,tx}$ to $u_{g,rx}$ by finding the shortest path over set of active links.

Link scheduling constraints

Link scheduling constraints for two types of TDD systems:

- Single-RU (radio unit) node case: in each T_k each u should have at most one active link with any $v \in T_k | v \neq u$, so that both (u, v) and (v, u) can be active
- Multi-RU node case: in each T_k each u should have either in-links or out-links only with any $v \in T_k | v \neq u$

Optimization problem as Steiner tree problem

To search for optimal solution, find a set of spatial links $\in V_H$ so that H provides optimal paths a) from u_g^{tx} to all u_i^{rx} and b) from all u_i^{tx} to u_g^{rx} fulfilling link scheduling constraints. Here, we assume that optimal paths can collated in to a tree and thus the problem becomes Steiner tree problem.

For general Steiner tree problem in undirected graphs, QUBO formulation is given in [3]. However, here we have directed graph and fixed root vertex, which allows us to give simplified problem formulation that is based on counting in- and out-edges (inspired by [2]). To provide paths to both upstream and downstream traffic (due to link scheduling, paths are not identical) we have to find both in-tree and out-tree. The problem formulation for out-tree is following (in-tree formulation is similar with reversed link directions):

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in E_H} w_{i,j} x_{i,j} \\
\text{s.t.} \quad & \sum_j x_{r,j} \geq 1 && \text{(root out-degree)},\\
& \sum_j x_{j,t} = 1, v_t \in S_d^{rx} && \text{(destination in-degree)},\\
& \sum_j x_{j,i} \leq 1, v_i \notin \{v_r\} \cup S_d^{rx} && \text{(others in-degree)},\\
& \sum_k x_{i,k} - \sum_j \sum_k x_{j,i} x_{i,k} = 0, v_i \notin \{v_r\} \cup S_d^{rx} && \text{(others if out-edge, needs in-edge)},\\
& \sum_j x_{i,j} - \sum_j x_{j,i} \geq 0, v_i \notin \{v_r\} \cup S_d^{rx} && \text{(others out-degree} \geq \text{in-degree)},\\
& \sum_i \sum_j x_{j,i} x_{i,j} = 0 && \text{(no in and out edges btw pairs)}
\end{aligned}
$$

Link scheduling constraint formulation for any $v_i \in T_k$ (left single-RU, right multi-RU):

$$
\sum_{j\in T_k} x_{i,j} \leq 1 \qquad \sum_{j\in T_k} x_{j,i} \leq 1 \qquad \sum_{j\in T_k} \sum_{k \neq j} x_{i,j} x_{k,i} = 0
$$

$$
\sum_{j\in T_k} \sum_{k\in T_k} x_{i,j} x_{k,i} = 0
$$

We are working in joining both out-tree and in-tree optimization with link constraints. Furthermore, 1st iteration of QUBO formulation is close to completion. We have validated the optimization problem formulation with Julia JuMP using EAGO optimization package.

Complexity estimate

Considering a single Steiner tree and assuming average vertex degree of 3, each vertex would have 5 edges in total (out-links, time link, and link to destination terminal). Furthermore, there are L copies of G and thus total number of edges should be $5|V(G)|L$ and so for both trees $10|V(G)|L$. Thus the search space should be order of $2^{10|V(G)|L}$.

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Unitary Zero Space

Cumucore

VTT

Summary

- **We have formulated the joint link scheduling and routing optimization problem as finding a rooted directed Steiner tree on a time-space graph.**
- **Optimization problem formulation with link scheduling constraints tested with Julia, JuMP and EAGO.**

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